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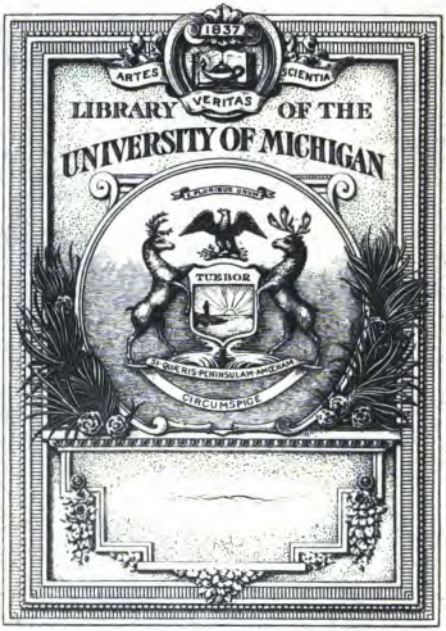
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THE
DESCRIPTION, NATURE and GENERAL USE,
OF THE
SECTOR
AND
PLAIN-SCALE,

Briefly and plainly laid down.

AS ALSO A

SHORT ACCOUNT
OF THE
USES
OF THE
Lines of Numbers, Artificial Sines and Tangents.

⁂

By Edmund Stone F. R. S.

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THE PREFACE.

Something concerning the Deſcription and Uſe of the Sector *and* Plain-Scale *being at this time very much wanted; and ſo I preſume will be very acceptable: therefore I have compiled this ſmall Treatiſe of them, in which is briefly contained, their* Deſcription, Nature, *and* General Uſe.

In the firſt Chapter *is contained, not only the Deſcription of the Lines upon the* Plain-Scale, *but likewiſe their Conſtruction and Nature are therein ſhewn: As alſo a ſhort Account of their Uſe.*

In the ſecond Chapter *is contained, the Deſcription of the* Sector, *and the Lines now commonly placed thereon.*

In the third Chapter *is ſhewn, the Ground and General Uſe of the* Sector. *From this* Chapter *it appears, that the* Sector *ſerves as a* Scale *to all Radius's; not greater than its Length, when quite opened; or leſſer than the Diſtance from 90 to 90 of the Sines, 60 and 60 of Chords, or 45 and 45 of Tangents, when the* Sector *is quite ſhut: And ſo by this Inſtrument* *the Sphere may readily be projected, upon the Plan of any Circle whoſe Radius not exceeds the afore-preſcribed Limits. Whence the Excellency of this Inſtrument above the* Plain-Scale *manifeſtly appears, for that is a* Scale *but to one Radius; and therefore by* it, *it is not eaſie to project the Sphere, unleſs upon a Circle whoſe Radius is of a given Length.*

In the fourth Chapter, *you have the chief particular Uſes of the Lines of Lines, and the Lines of Polygons.*

In the fifth Chapter, *is ſhewn the manner of working Proportions with Lines and Sines, Sines and Tangents, Lines and Tangents,* &c. *As alſo the excellent Uſe of the* Sector *in drawing the Hour Lines upon an horizontal and erect* South *Plan.*

Laſtly, in the ſixth Chapter *is contained, the Uſe of the Lines of Numbers, artificial Sines and Tangents. What is ſaid in this* Chapter, *I am of Opinion, is ſufficient to ſhew Perſons indifferently ſkilled in Arithmetick and Geometry the Manner of uſing thoſe Lines.*

Thus, courtious Reader, have I given you the Contents of what is to be ſeen in the following little Tract, hoping you will candidly receive the ſame. And ſo I remain, yours, &c.

E. S.

THE
Deſcription, Nature and General Uſe,
OF THE
Sector *and* Plain-Scale, *&c.*

CHAP. I.

Concerning Definitions of Chords, Sines, Tangents, &c. *and the Manner of Projecting them, and putting them on Rulers, commonly call'd Plain-Scales.*

Becauſe of the ſix Parts of a Triangle, *viz.* the three Sides and the three Angles, any three of them being given, independent of one another, the other three are thereby limited, and conſequently ſome way or other to be found. And ſince the Angles of a Triangle, that is, the Arcs of equal Circles deſcribed about the angular Points, and comprehended between the Sides, (which are proportional to the Angles, from *prop.* 33. *lib.* 3.

Euclid) are not proportional to its Sides, the Ancients deviſed certain right Lines, appertaining to a Circle, which come in competition with Angles or Arcs. Theſe right Lines are thus defined;

Fig. 1.

A Chord of an Arc or Angle, is a right Line drawn from one end of an Arc to the other; as the Line AB is the Chord of the Arc AB; it is alſo the Chord of the Arc AFB, becauſe it is common to both Arcs of the Circle.

Whence it is manifeſt, (*per prop.* 15. *lib.* 3. *Euclid*) that the greateſt Chord that can be drawn in a Circle is its Diameter, or the Chord of 180 Degrees, or a Semicircle: Whence all Chords of Arcs, greater than a Semicircle, are leſſer than the Diameter.

A right Sine of an Arc, is a right Line drawn from one end of that Arc, perpendicular to a Diameter drawn to the other end of it, and belongs to both Arcs of a Semicircle; as BC is the right Sine of the Arc AB, and alſo of the Arc FB. Hence it is manifeſt, from the aforecited Propoſition of *Euclid*, that the greateſt Sine is equal to the Semidiameter GE, or the Sine of 90 Degrees: Therefore all Sines of Arcs greater than 90 Degrees, or a Quadrant, are leſſer than the Semidiameter, or Radius.

The verſed Sine of an Arc, is that Portion of the Diameter included between the right Sine of the ſaid Arc, and the Arc it ſelf. As AC is the verſed Sine of the Arc AB; and FC the verſed Sine of the Arc FGB; hence the greateſt verſed Sine is the Diameter AF, *viz.* the verſed Sine of 180 Degrees, or a Simicircle.

A Tangent of an Arc, is a right Line touching the Circumference of a Circle, and (from *prop.* 16. *lib.* 3. *Euclid*) is at right Angles to the Diameter drawn thro' the Point of Contact, and limited by the Secant of the ſame Arc; as AD is the Tangent of the Arc AB; whence there can be no Tangent of 90 Degrees, or a Quadrant, for then the Tangent will be infinitely long.

The Secant of an Arc, is a right Line drawn from the Center of a Circle, thro' one end of the Arc it is the Secant of, till it meets the Tangent, raiſed at right Angles to a Diameter drawn to the other end of the Arc; as ED is the Secant of the Arc AB; whence there can be no Secant of 90 Degrees, or a Quadrant, becauſe then the Secant will be infinitely long.

The Half or Semitangent of an Arc, is that Portion, next to the Center, of a right Line drawn from the Center of the Circle parallel to the Tangent of the Arc, cut off by the Chord of the Complement of the Arc to 180 Degrees, as EH is the half Tangent of the Arc AB. But becauſe (from *prop.* 20. *lib.* 3. *Euclid*) the Angle BFA is ½ of the Angle BEA, the Side FE equal to the Side EA, and the Angles EAD, FEH right ones, HE will be equal to the Tangent of half the Arc BA; that is, the

Semitangent of any Arc is but the Tangent of half that Arc; whence the greateſt Semitangent, or that of 180 Degrees, is infinite.

The Circumference of every Circle is ſuppoſed to be divided into 360 equal Parts, called Degrees; and each Degree in 60 equal Parts, called Minutes, *&c.* This Number was choſen by Geometricians for the Diviſion of a Circle, becauſe it may be divided into a greater Number of Parts without any Remainder, than any leſſer Number than 360.

The Complement of an Arc, or Angle, is what it wants of a Quadrant, or 90 Degrees, or of a Semicircle, or 180 Degrees; or laſtly, of a whole Circle, or 360 Degrees. As 20 Degrees is the Complement of 70 Degrees to a Quadrant, becauſe 20 Degrees is the Remainder of 70 Degrees ſubſtracted from 90 Degrees, likewiſe 50 Degrees is the Complement of 130 Degrees to 180 Degrees, and 70 Degrees the Complement of 290 Degrees.

How to Project the Plain-Scale.

Fig. 2. First draw a Circle, ABDC, which croſs at right Angles with the Diameters AD, CB; then continue out AD to G, and upon the Point B raiſe BF perpendicular to CB. Now draw the Chord AB, and divide the Quadrant AB into 9 equal Parts, ſetting the Figures 10, 20, 30, *&c.* to 90, to them; each of which 9 Parts again ſubdivide into 10 equal Parts, and then the Quadrant will be divided into 90 Degrees. Now ſetting one Foot of your Compaſſes in the Point A, transfer the ſaid Diviſions to the Chord Line AB, and ſet thereto the Figures 10, 20, 30, *&c.* and the Line of Chords AB, will be divided, and then may be put upon your Scale.

Now to project the Sines, divide the Arc BD into 90 Degrees, as before you did AB; from every of which Degrees let fall Perpendiculars on the Semidiameter EB, which Perpendiculars will divide EB into a Line of Sines, to which you muſt ſet the Numbers 10, 20, *&c.* beginning from the Center, and then you may transfer the Line of Sines to your Scale.

Again, to project the Line of Tangents, from the Center E, and thro' every Diviſion of the Arc BD, draw right Lines cutting BF, which will divide it into a Line of Tangents, ſetting thereto the Numbes 10, 20, 30, *&c.* which you muſt transfer to your Scale.

To project the Line of Secants, transfer the Diſtances E 10, E 20, E 30, *&c.* that is, the Diſtance from E to 10, 20, 30, *&c.* on the Tangent Line, upon the Line EG, and ſetting thereto the Numbers 10, 20, 30, *&c.* the Line EG will be divided into a Line of Secants.

To project the Semitangents, draw Lines from the Point C thro' every Degree of the Quadrant AB, and they will divide the Semidiameter AE into a Line of Semitangents: But becauſe the Semitangents on Scales run to 160 Degrees, continue out the Line AE, and draw

Lines from the Point C, thro' the Degrees of the Quadrant CA, cutting the ſaid continued Portion of AE, and you will have a Line of half Tangents to 160 Degrees, or farther if you pleaſe.

Moreover to draw the Rhumb Line, from every 8*th* Part of the Quadrant AC, ſetting one Foot of your Compaſſes in A, deſcribe Arcs cutting the Chord AC, which will divide AC into a Line of whole Rhumbs; and after the ſame manner may the Subdiviſions of half and quarter Rhumbs be made.

Laſtly, to project the Line of Longitude, draw the Line HD equal and parallel to the Radius CE, which divide into 60 equal Parts, every 10 of which Number. Now from every of thoſe Parts let fall Perpendiculars to CE, cutting the Arc CD, and having drawn the Chord CD with one Foot of your Compaſſes in D, transfer the Diſtances from D to each of the Points in the Arc CD, on the Chord CD, and ſet thereto the Numbers 10, 20, *&c.* and the Line of Longitude will be divided.

Theſe be all the Lines commonly put on one ſide of the Plain-Scale, excepting equal Parts, which want no deſcription: And on the other ſide is a Decimal or Diagonal Scale, on which an Inch, or ſome part thereof, as $\frac{1}{2}$, or $\frac{1}{4}$, is divided into 100 equal Parts, by means of Diagonals, in the following manner;

Fig. 3. Suppoſe AB to be the Length of the Scale, which let be 6 Inches; then having choſen a convenient Breadth, as BC; make the Parallelogram AEBC, and draw Lines from the Diviſion of every Inch, parallel to BC; then divide an Inch, ſuppoſe DB, into 10 equal Parts, as alſo FC, and draw the Diagonals D 10, 10 20, 20 30, *&c.* Likewiſe divide BC, and AE, each into 10 equal Parts, thro' which draw 10 Parallels, and ſet the Numbers as *per* Figure, and your Scale will be made. After the ſame manner may one half or one quarter of an Inch be decimally divided.

Now if you have a mind to take any part of an Inch, ſuppoſed to be divided into 100; as the 48*th* part, look for the Diviſion 40 on the Line DB, and guiding your Eye along its Diagonal, mark where the 8*th* Parallel to BD cuts the ſaid Diagonal, and the Diſtance of that Point of Section from the Line DF, will be the 48*th* part of an Inch. Again, to find the 100*th* part of an Inch, look where the Parallel 1 cuts the Diagonal D 10, and the Diſtance from that Point to DF will be the 100*th* part of an Inch. For from the Similiarity of Triangles, demonſtrated *per prop.* 4. *lib.* 6. *Euclid*, as DF is to $\frac{1}{10}$ of DB, or 10 F, ſo is $\frac{1}{10}$ of DF to $\frac{1}{100}$ of DB.

Hence it is manifeſt, that by means of this Scale, any Line may be drawn whoſe Length is expreſſible in leſs than four Figures, whether they be whole Numbers, or mixed, that is decimal Fractions, and whole Numbers; as if the Length of the Line be 365; then you muſt call 1 Inch 100, and ſo 3 Inches will be 300. Then the 65 muſt be taken from DF, to the Interſection of the *60th* Diagonal with the *5th* Parallel, and the Extent from 3 Inches to that Point of Interſection, will give the Length of the Line expreſſible by 365. Likewiſe the ſame Extent may be taken for the Length of a Line expreſſible by 36.5, in ſuppoſing 1 Inch to be 10. Alſo it may be expreſſed by 3.65, in taking 1 Inch for 1. And laſtly, it may be expreſſed by .365, in taking 1 Inch for $\frac{1}{10}$. And this is the Uſe of the Diagonal Scale.

As for the Uſes of the Chords, Sines, Tangents, *&c.* the Chords are to lay off the Quantity of any Angle deſired upon a Point given, in a right Line. And contrarywiſe, to meaſure the Quantity of an Angle already laid down. The firſt is done by always taking the Extent of 60 Degrees of Chords between your Compaſſes, and deſcribing an Arc about the angular Point; and then laying off the Number of Degrees propoſed, upon the ſaid Arc, and drawing a right Line from the angular Point. And the latter, by always making an Arc of 60 Degrees of Chords about the angular Points, and then taking the Chord of the ſaid Arc between your Compaſſes, and meaſuring it on the Line of Chords.

Example, To make an Angle of 30
Fig. 4. Degrees on the Point A, take 60 Degrees of Chords in your Compaſſes, and ſetting one Foot of your Compaſſes in the Point

Point A, deſcribe the Arc DC; then take 30 Degrees from your Chords, and lay them off from D to C, and draw the Line AC, and the Angle CAB will be 30 Degrees.

Now to meaſure an Angle, ſuppoſe CAB, take 60 Degrees of Chords between your Compaſſes, and ſetting one Foot in the Point A, deſcribe the Arc CD; then take between your Compaſſes the Diſtance from C to D, which meaſur'd on the Chords will be found to reach to 30 Degrees, the Quantity of the Angle ſought.

The Sines are to othographically project the Sphere, *&c.*

The Tangents, half Tangents and Secants, are uſed in finding the Centers, and Poles, of projected Circles, in the Stereographical Projection of the Sphere, *&c.*

The Rhumbs, are to lay down the Angles of a Ship's Way in Navigation.

And the Line of Longitude determines by Inſpection, how many Miles there are in a Degree of Longitude, in each ſeveral Latitude: As, in the Latitude of no Degrees, that is, under the Equator, 60 Miles make a Degree; in the Latitude of 40 Degrees, 46 Miles make a Degree; in the the Latitude of 60 Degrees, 30 Miles make a Degree; in the Latitude of 80 Degrees, 10 Miles make a Degree.

CHAP. II.

The Deſcription of the Sector, *and the Lines generally placed thereon.*

Fig. 24. THE *Sector* is a Mathematical Inſtrument, conſiſting of two equal Legs, of Braſs, Silver, Ivory or Wood, moveable about a Joynt, like a Carpenters Rule; ſo that the ſaid Legs, together with certain right Lines, drawn from the Center of the Joynt, contains Angles of different Magnitudes. The Length of theſe Legs are not determined, but yet they are generally about 6 Inches, having a proper Breadth and Thickneſs. This Inſtrument takes its Name, from a Sector in Geometry, which *Euclid* defines, in the *9th* Def. of his *3d* Book, to be a Figure comprehended under two right Lines making an Angle, and the Arc of a Circle deſcribed about the angular Point, intercepted between the two ſaid right Lines.

The right Lines that be commonly drawn on the Faces of this Inſtrument, to be uſed Sectorwiſe, are ſix, *viz.* The Lines of Lines, or equal Parts, the Lines of Chords, the Lines of Sines, the Lines of Tangents, the Lines of Secants, and the Lines of Poligons.

The Lines of equal Parts are two equal right Lines drawn upon the Legs of the Sector, on the ſame Side, from the Center of the Joynt, almoſt to the Ends of the Legs, making unequal Angles with the inward Edges of their reſpective Legs; that ſo, the Lines of Sines, and Secants, which are likewiſe drawn upon the ſame Faces of the Legs from the Center of the Joynt, may make equal Angles with each other, and likewiſe equal to that which the ſaid Lines of Lines make. Theſe Lines of Lines, are each divided into 100 equal Parts, each of which are again ſubdivided into Halfs, and Quarters. If the Length of the Legs, are ſufficient. The Diviſions are number'd, from the Center, with 1, 2, 3, *&c.* to 10. near which are placed the Letters LL. The three Parallels, that be drawn, to each of theſe Lines, are for the Eyes better diſtingiſhing the Diviſions, and Subdiviſions, as alſo are thoſe near the other Lines.

The Lines of Sines, are two equal Lines of natural Sines, drawn upon the ſame Faces of the two Legs of the Sector from the Center of the Joynt, as the Lines are, of a Radius, equal in Length to either of the Lines of Lines. Theſe are number'd with the Figures 10, 20, *&c.* to 90. at the Ends of which ſtand the Letters SS.

The Lines of Chords, on the other Faces of the ſame Legs, are two natural Lines of Chords (made as directed in the laſt Chapter) drawn from the Center of the Joynt upon each Leg, ſo as to make an Angle with each other, equal to the Angle that the Lines of Sines, *&c.* make. The Radius of theſe Lines of Chords is equal in length to the Radius of the Lines of Sines. They are number'd with 10, 20, *&c.* to 60

Degrees; the Chord of which, *per Corol. prop.* 15. *lib.* 4. *Euclid*, is equal to the Radius. Theſe Lines are denoted by the Letters CC.

The two Lines of Tangents, are Lines of natural Tangents, (divided as directed in the laſt Chapter) whoſe Radius is equal in length to the Radius of the Lines of Sines. Theſe are drawn from the Center of the Joynt on each Leg, making the ſame Angle with each other, as the Sines, *&c.* do. Each of theſe Lines of Tangents run but to 45 Degrees, and are number'd accordingly. The reaſon of this is, becauſe the Tangent of 45 Degrees is equal to Radius; that ſo when the Sector is ſet to 90 Degrees on the Lines of Sines, the ſame Extent, that is, from 90 Degrees on one Line of Sines to 90 Degrees on the other, will reach from 45 Degrees to 45 Degrees on the Lines of Tangents; as alſo, from 60 Degrees to 60 Degrees, on the Lines of Chords. The Lines of Tangents are marked with the Letters TT.

There are alſo two Lines of leſſer Tangents, whoſe Radius is about two Inches in length, drawn from the Center of the Joynt, and making an Angle with each other, equal to the Angle that any one of the aforeſaid Lines make reſpectively with each other; tho' on ſome Sectors this Angle is leſſer. Theſe Lines of Tangents are number'd from 45 Degrees, which ſtands at the Ends of the Radius from the Center of the Joynt, to about 75 Degrees.

They have the Letters tt or tan ſet to them; and ſerve to ſupply the Defect in the other Lines of Tangents.

The Lines of Secants, are divided from a Circle of the ſame Radius as the Lines of leſſer Tangents are, and make the ſame Angle with each other as the leſſer Tangents do. Theſe begin at the Length of the Radius from the Center, and run to about 75 Degrees. The Numbers 10, 20, 30, &c. to 75, are ſet to them, and they are known by the Letters SS. or sec

Laſtly, The Lines of Polygons, are drawn from the Center of the Joynt very near the inward Edges, and number'd from 4, 5, &c. to 12, which is about 3 Inches from the Center. Theſe Lines are denoted with the Letters PP.

Thus have I deſcribed all the Lines that be commonly drawn on a Sector to be uſed Sectorwiſe.

The other Lines that be arbitrarily placed, near and parallel to the outward Edges of the Sector, on both Faces thereof, and which are to be uſed, as on common Scales, are the Lines of artificial Numbers, Sines and Tangents; a Foot divided into 12 Inches, and each Inch ſubdivided into 20 equal Parts. Alſo ſometimes the Lines of Latitude, Hours, and Inclination of Meridians are put on it.

Note, The artificial Numbers, are denoted with the Letter N, the Sines with the Letter S, and the Tangents with the Letter T. When Proportions are to be work'd, by theſe artificial Lines, the Sector muſt be quite open'd.

CHAP. III.

Of the Foundation and General Uſe of the Sector.

BEcauſe, from *Prop.* 4. *lib.* 6. *Euclid*, the homologus Sides of ſimilar or equiangular Triangles, are proportional to each other; therefore if the Lines AB, AC, repreſent, for example, the two Lines of Lines, making any Angle BAC with each other. Now if the Line BC be drawn, and Aa, Ab, taken equal, and the Line ab drawn, AB, or AC, will be to BC, as Aa is to ab. Whence if the Line AB or AC be double, triple, quadruple, *&c.* the Line Aa or Ab, the Line BC will be double, triple, quadruple, *&c.* of the Line ab. As ſuppoſe AB, or AC, be 100, and Aa, or Ab 50, and BC 60, then ab will be 30; for as AB 100 is to Aa 50, ſo is BC 60 to ab 30. And the like Reaſon holds for all other Proportions.

Fig. 5.

Whence if the Lines AB, AC, repreſent the Lines of Chords, Sines or Tangents; that is, if the Lines AB, or AC, be the Radius, and the Line Aa is the Chord, Sine or Tangent of a propoſed Number of Degrees, to the ſaid Radius AB, the Line ab will be the Chord, Sine or Tangent of the ſame Number of Degrees to the Radius BC.

For example, To find the Chord, Sine or Tangent of 15 Degrees, to a Radius of 4 Inches. Open the Sector ſo, that the Diſtance from *60* to *60* on the Lines of Chords be 4 Inches; then the Diſtance between 15 and 15 on the Lines of Chords, will be the Chord of 15 Degrees to the Radius of 4 Inches. The Sector ſtill remaining thus opened, the Extent from 15 Degrees to 15 Degrees on the Lines of Sines, will be the Sine of 15 Degrees, to the aforeſaid Radius of 4 Inches. Alſo the Extent from 15 Degrees to 15 Degrees on the Lines of Tangents, will give the Tangent of 15 Degrees to the aforeſaid Radius.

But if the Chord propos'd be more than *60* Degrees, as ſuppoſe the Chord of 80 Degrees to the aforeſaid Radius be requir'd, you muſt take, as before directed, the Chord of half 80 Degrees, ~~*viz.* 40 Degrees;~~ *Fig.* 6. and having deſcribed an Arc, AD, with the ſaid Radius, lay off the Chord of 40 twice; to wit, from A to B, and from B to C; draw the Line AC, which will be the Chord of 80 Degrees.

If the Tangent of a Number of Degrees greater than 45, ſuppoſe 74 Degrees, be ſought to a Radius of $3\frac{1}{2}$ Inches, you muſt make uſe of the Lines of leſſer Tangents; becauſe the greater ones run but to 45 Degrees, thus. Open the Sector ſo, that the Diſtance between

45 Degrees and 45 Degrees of the leſſer Tangents, be equal to 3½ Inches; then the Diſtance from 74 Degrees to 74 Degrees, will be the Tangent of 74 Degrees to a Radius of 3½ Inches.

If the Secant of a propoſed Number of Degrees to a given Radius be ſought; as ſuppoſe the Secant of 50 Degrees to a Radius of 1 Inch be required, open the Sector ſo, that the Diſtance between 0 and 0, on the Lines of Secants be 1 Inch; then the Diſtance from 50 Degrees to 50 Degrees on the ſame Lines, will be the Secant of 50 Degrees.

If the Converſe of any of theſe Operations be requir'd; that is, if a given right Line be the Chord, Sine, Tangent or Secant of a propoſed Number of Degrees, ſuppoſe 16, and the Radius to it be ſought, open the Sector ſo, that the Diſtance between 16 and 16, on the Lines of Chords, Sines, Tangents or Secants, be equal to the Length of the given Line, according as it be a Chord, Sine, Tangent or Secant. Then the Diſtance between 60 Degrees and 60 Degrees on the Lines of Chords; between 90 Degrees and 90 Degrees on the Lines of Sines; betwen 45 Degrees and 45 Degrees on the Lines of Tangents; and between 0 Degrees and 0 Degrees on the Lines of Secants, will be the Radius to the ſaid right Line of 16 Degrees, according as it is a Chord, Sine, Tangent or Secant.

From what hath been already ſaid, it appears that the Sector is as it were a univerſal

Plain-Scale; for whereas on the Plain-Scale, the Chords, Sines, Tangents and Secants, have but one Radius; ſo by this Contrivance, the Chords, Sines, Tangents or Secants to all Radius's may be had. But with this Limitation, that the Radius given be not longer than the Length of twice the Radius of the reſpective Chords, Sines, Tangents or Secants, on the Legs of the Sector; or ſhorter than the Diſtance from 60 to 60 of Chords, 90 and 90 of Sines; or 45 and 45 of Tangents, when the Sector is quite ſhut.

Moreover, Becauſe the Lines of Lines, Chords, Sines, Tangents and Secants, make equal Angles with each other reſpectively, therefore Proportions in equal Parts and Sines, equal Parts and Tangents, Sines and Tangent, *&c.* by help of a Pair of Compaſſes, and the Sector, may be work'd. And ſo the Caſes of Plain and Spherical Trigonometry may, by means of the Sector, be ſolved. An Inſtance or two of which I ſhall give, after having given ſome of the particular Uſes of the Lines of Lines, and the Uſe of the Lines of Polygons.

* * *

CHAP. IV.

Of the particular Uſes of the Lines of Lines; and the Uſe of the Lines of Polygons.

Fig. 5. FOR the Eaſe of Expreſſion, in ſhewing the Uſe of the Lines on the Sector, the Lines found out by the Sector are ſaid to be lateral or parallel; lateral are thoſe found on the Sides of the Sector, as AB, AC; and parallel are the Lines that run from one Leg of the Sector to the other, equally diſtant from the Center of the Joynt; as BC, ab.

PROB. I.

To lay down a Line expreſſable by any given Parts, or Fractions of Parts.

Either of the Lines of Lines are actually divided into 100 equal Parts, but we have put only 10 Numbers to them, which may ſignifie either themſelves alone, or 10 times themſelves, or 100 times themſelves, or 1000 times themſelves, as occaſion requires. As ſuppoſe the Numbers given be no more than 10, then we may ſuppoſe the Lines only divided into 10 Parts, according to the Numbers ſet to them. If they be more than 10, but leſſer than 100, then either Line will contain 100 Parts, and the Numbers ſet by them will be in value, 10, 20, 30, &c. as they are actually divided. If yet they be more than 100, then every Part muſt be ſuppoſed divided into 10, and either Line will be 1000 Parts, and the Numbers ſet to them will be in value 100, 200, 300, and ſo forwards, ſtill encreaſing themſelves by 10. This being pre-ſuppoſed, we may number the Parts, and Fraction of Parts given on either of the Lines of Lines; which Diſtance taken from them with a Pair of Compaſſes, and drawn on your Paper, will give the Length of the Line ſought.

Example. Suppoſe a Line is to be drawn repreſenting 75; take 75 of the *Fig.* 7. 100 Parts either of the Lines of Lines is divided into, and note it on A; or take 7½ of the firſt 10 Parts, and note it on B; or only take ¾ of thoſe 100 Parts, and note it in C; either of which will be Lines expreſſable by 75.

PROB. II.

To Encreaſe, or Deminiſh, a Line in a given Proportion.

Take the Line given between your Compaſſes, and open the Sector, ſo as the Feet of the Compaſſes may ſtand in the Points of the Number given on each of the Lines of Lines; then keeping the Sector thus opened, the parallel Diſtance of the Points of the Number required, will give the Line ſought.

Example. Let A be a Line given to be encreaſed in the Proportion of 3 to *Fig.* 8. 5. Firſt take the Line A between your

Compaſſes, and open the Sector ſo, that the Diſtance from 3 to 3 on the Lines of Lines be equal to that Extent; then the parallel Diſtance of 5 and 5, will give the Line B, ſought.

In the ſame manner, if B be a Line given to be diminiſhed in the Proportion of 5 to 3, take the Line B between your Compaſſes, and to it open the Sector in the Points of 5 and 5, on the Lines of Lines; then the parallel Diſtance between 3 and 3, will give the Line A, required.

If this way of working be not ſufficient, you may multiply the Numbers given by 2, 3, 4, &c. and work by their Equimultiples; as, for 3 and 5, you may open the Sector in 6 and 10, 9 and 15, 12 and 20, 15 and 25, or in 18 or 30, &c.

PROB. III.

To divide a given Line into any Number of equal Parts.

Take the Line given between your Compaſſes, and open the Sector in the Points of the Parts the given Line is to be divided into; then keeping the Sector at this Angle, the parallel Diſtance between the Points of 1 and 1, will divide the given Line into the Parts required.

Example. Let AB be a Line given *Fig. 9.* to be divided into 8 equal Parts. Firſt take this Line AB between your Compaſſes, and to it open the Sector in the Points of 8 and 8; then the parallel Diſtance between

1 and 1, gives the Line AC, which will divide AB into 8 equal Parts.

If the Line propoſed be too long to be applied to the Legs of the Sector, divide ½ or ¼ of it into the Parts propoſed; and the Double, or Quadruple of one of thoſe Parts, will divide the whole Line into the Number of Parts propoſed.

PROB. IV.

To find the Proportion between two or more given Lines.

Take the greater of the given Lines between your Compaſſes, and to it open the Sector in the Points of 100 and 100; then take the leſſer Lines ſeverally, and carry them parallel to the greater, till they ſtay on the ſame Numbers of the Lines of Lines; and the Numbers whereon they ſtay, will give their Proportion to 100.

Example. Let the Lines given be AB, CD. Firſt take the Line CD *Fig.* 10. between you Compaſſes, and to it open the Sector in the Points of 100 and 100; then keeping the Sector thus opened, enter the leſſer Line AB parallel to the former, and you will find it croſs the Lines of Lines in the Points 60 and 60; wherefore the Proportion of AB to CD, is as 60 to 100.

Or, if the Line CD be ſo long that it cannot be apply'd to the Points 100 and 100, then you may ſuppoſe the leſſer Line AB, to be 100, and cutting off CE equal to AB, you will find the Proportion of CE to ED, to be as 100 to almoſt 67; wherefore this way the Proportion of AB to CD, is as 100 to almoſt 167.

This Proportion may likewiſe be worked by any other Numbers that admit ſeveral Diviſions, as by the Numbers 60 and 60; and then the leſſer Number will be found 36, which is as before in leſſer Numbers, as 3 to 5. This Proportion may alſo be worked without opening the Sector. For if the Lines between which a Proportion be ſought, are laterally apply'd on the Lines of Lines, (or any other Scale of equal Parts) the Proportion between them will be the ſame, as between the Lines to which they are equal.

PROB. V.

To find a Third Proportional to two given Lines.

Firſt place both the given Lines on both ſides of the Sector from the Center, and mark the Terms of their Extenſion; then take out the ſecond given Line again, and to it open the Sector in the Term of the firſt Line; then keeping the Sector at this Angle, the parallel Diſtance between the Terms of the ſecond Line, will be the third Proportional ſought.

Fig. 11. Example. Let the two Lines given be AB, AC, which take out and place on both ſides of the Sector, ſo as they meet in the Center A: let the Terms of the firſt Line be B, B, and the Terms of the ſecond C, C. Then take out AC, the ſecond Line, again, and to it open the Sector in the Terms B, B; ſo the Parallel between C, C, gives the third Proportional ſought. For as AB is to AC, ſo is BB equal to AC, to CC.

PROB. VI.

Three Lines being given, to find a fourth Line proportional to them.

Place the firſt and third Lines on both ſides of the Sector from the Center; then take out the ſecond Line, and to it open the Sector in the Terms of the firſt Line. Now keeping the Sector at this Angle, the parallel Diſtance between the Terms of the third Line, will be the fourth Proportional ſought.

Fig. 12.

Example. Firſt take out A and C, and place them on both ſides of the Sector, in AB, AC, and AD, AE, laying the beginning of both Lines in the Center A; then take out B, the ſecond Line, and to it open the Sector in B, C, the Terms of the firſt Line; and then the Parallel between D and E, gives the fourth Proportional ſought.

As in Arithmetick, it is ſufficient for the firſt and third given Numbers, to be of one Denomination, and the ſecond and the fourth required to be of another. For one and the ſame Denomination is not neceſſarily required in them all. So in Geometry, it is ſufficient if the Sides AB, AD, reſembling the firſt and third Lines be meaſured by one Scale, and the Parallels BC, DE, be meaſured in another. Wherefore the Proportion of A, the firſt Line to C the third, being found by *Prop.* 4. aforegoing, which ſuppoſe to be as 8 to 12, or in leſſer Numbers, as 4 to 6, or as 2 to 3, or in greater Numbers, as 16 to 24, or 18 to 27, or 20 to 30, 30 to 45, or 40 to 60, *&c.* If the Sector be opened in the Points of 8 and 8, to the Quantity of B, the ſecond given Line; then the Parallel between 12 and 12, will give DE, the fourth Line required. So likewiſe if it be opened in 4 and 4, the Parallel between 6 and 6; or if in 16 and 16, the Parallel between 24 and 24, will give the ſame, DE, and ſo of the reſt.

PROB. VII.

Two Lines being given, to find a Mean Proportional between them.

Take the longeſt Line and lay it on the Line of Lines from the Center, noting its Extention; then take the ſhorteſt Line between your Compaſſes, and applying it parallelwiſe on the Lines of Lines, open the Sector ſo, that the lateral Diſtance from the Center of the Sector to the Term of this Line, be equal to the parallel Extent of the Terms of the longeſt Line; and then the ſaid lateral Diſtance of the Terms of the ſhorteſt Line, or the parallel Diſtance of the Terms of the longeſt Line, will give the Mean Proportional ſought.

Note, This cannot be done unleſs by ſeveral Tryals of opening the Sector, and moving the Points of the Compaſſes.

Example, Let A and B, be the two given Lines. Lay B from the Center A of the Sector, which ſuppoſe to reach to B and B; then take A in your Compaſſes, and open the Sector ſo that DD being equal to A, AD may be equal to BB. And then AD or BB will be a Mean Proportional between the Lines A and B; for as AB is to BB, ſo is AD, equal to BB, to DD. *Fig.* 13.

PROB. VIII.

To divide a Line in the ſame Manner as another Line is already divided.

First, Take the given Line already divided, and laying it on both Sides of the *Sector* from the Center, note how far it extendeth. Then take out the ſecond Line, which is to be divided, and to it open the *Sector* in the Terms of the firſt Line. This done, take out the Parts of the firſt Line, and place them alſo on the ſame Side of the *Sector* from the Center. And the Parallels taken in the Terms of theſe Parts, will be the correſpondent Parts in the Line to be divided.

Example. Let AB be a Line divided in D and E, and BC the Line to be divided in ſuch manner as AB is. *Fig.* 14.

Firſt, Take out the Line AB, and place it on the Lines of Lines in AB, AC, both from the Center A; then take out the ſecond Line BC, and to it open the Sector in B and C, the Terms of the firſt Line. The Sector remaining thus opened, take out AD, and AE, the Parts of the firſt Line AB, and place them alſo on both ſides of the Sector in AD, AE; and the Parallel DD, gives BF, and the Parallel EE, BG. Now the Line BC, is divided in F and G, as the Line AB, is in D, E; which is the thing requir'd.

PROB. IX.

To open the Sector ſo, that the two Lines of equal Parts may make a right Angle.

Chuſe three Numbers that may expreſs the Sides of a right angled Triangle; as are, for example, the Numbers 3, 4, 5, or their Multiples; but becauſe it is better to take them bigger, chuſe 60, 80, and 100. Now take in your Compaſſes the Diſtance from the Center of the Sector on either of the Lines of Lines to 10, which let repreſent 100; then open the Sector ſo, that the Diſtance between 6 on one Line of Lines, and 8 on the other, (the one repreſenting 60, and the other 80) be equal to that Diſtance: And then the Sector will be opened ſo that the two Lines of Lines make a right Angle.

PROB. X.

To find a right Line equal to the Circumference of a given Circle.

The Diameter of a Circle to its Circumference, is as about 3.18 to 10; whence take the Diameter of the propoſed Circle between your Compaſſes, and ſet it over as a Parallel on 3.18 and 3.18 of the Lines of Lines. The Sector remaining thus opened, take the parallel Extent of 10 and 10, and that will give you near the Length of the Circumference ſought.

PROB. XI.

To inſcribe a regular Polygon in a given Circle.

Take the Radius of the given Circle between your Compaſſes, and ſet it over as a Parallel on 6 and 6 of the Lines of Polygons; then the parallel Diſtance of the Numbers repreſenting the Number of Sides a Polygon is to have, will be the Side of the requir'd Polygon.

Example. Suppoſe AGH be a Circle, in which it is required to inſcribe a Pentagon. Take the Radius AB in your Compaſſes, and ſet it over as a Parallel on 6 and 6 of the Lines of Polygons; then the parallel Diſtance of 5 and 5, on the ſaid Lines of Polygons, will give AC, the Side of a Pentagon to be inſcribed in the Circle. Whence it will be eaſie to make the Polygon in carrying the Diſtance AC round the Circle, and drawing 5 right Lines.

Fig. 15.

PROB. XII.

Upon a right Line given to make a regular Polygon.

Take the given right Line between your Compaſſes, and to it open the Sector in the Points of the Number of Sides the Polygon is to have; then the parallel Diſtance between 6 and 6, will give the Length of the Radius of a Circle, whoſe Center is the Vertex of an Iſcocles Triangle made with the aforeſaid Radius's upon the given Line. Now I ſay, if about the ſaid Center a Circle be deſcribed with the aforenamed Radius, and the aforeſaid given Line be carried round its Circumference, and right Lines drawn, that the propoſed Polygon will be made.

Fig. 16. Example. To make a Heptagon upon the given Line AB. Take the ſaid Line between your Compaſſes, and ſet it over as a Parallel on the Points of 7 and 7 of the Lines of Polygons; then the parallel Diſtance between 6 and 6 will give the Radius of a Circle. Now with this Radius ſetting one Foot in the Points A, B, the Extremes of the given Line, deſcribe two Arc's cutting each other in C, about which deſcribe the Circle ABD; then carry AB round the Circumference, and draw 7 right Lines, and the Heptagon will be made.

CHAP.

CHAP. V.

Of working Proportions in Lines and Sines, Lines and Tangents, Sines and Tangents, &c. exemplified, in the Reſolution of ſome of the Caſes of plain, and ſpherical Trigonometry.

PROB. I.

The Hypothenuſe, and Baſe of a right lined right angled Triangle being given, to find the Angle oppoſite to the Baſe.

SUppoſe the Length of the Hypothenuſe AB be 70 Miles, and the Length of the Baſe AC 40 Miles, what is the Quantity of the Angle ABC? *Fig.* 17.

As the Length of the Hypothenuſe AB is to Radius, ſo is the Baſe AC to the Sine of the Angle ABC.

Therefore to work this Proportion on the Sector, take the Extent from the Center on either of the Lines of Lines to the Number 7, which let repreſent the Length of the Hypothenuſe 70 Miles; then open the Sector, and ſet this Extent over as a Parallel on the Points of 90 and 90 on the Line of Sines. The Sector remaining thus opened, take the Length of the Baſe 40 Miles from the Line of Lines, and applying it parallelwiſe on the Sines, its Extremes will give 34 Degrees 50 Minutes for the Quantity of the Angle ABC ſought.

PROB. II.

Fig. 18. *The Baſe AC of the oblique angled Triangle ABC being given, ſuppoſe 60 Leagues; the Angle ABC, 130 Degrees, and the Angle ACB, 46 Degrees; to find the Length of the Side AB.*

As the Sine of the Angle ABC is to the Length of the Baſe AC, ſo is the Sine of the Angle ACB to the Length of the Side AB ſought.

To work this by the Sector, take the lateral Extent of 46 Degrees on either of the Lines of Sines; then open the Sector, and apply this Extent parallelwiſe in the Points of 50 and 50, (the Complement of the Angle ABC), on the Lines of Sines. The Sector remaining thus open'd, take the parallel Extent from 6 to 6, repreſenting 60 and 60, on the Lines of Lines, and meaſure it laterally on either of them, and you will find it to be about 56, which is the Length in Leagues of the ſought Side AB.

PROB. III.

Fig. 19. *The Perpendicular and Baſe of a right angled Triangle ABC being given, the firſt ſuppoſe 20 Yards, and the latter 60; to find the Quantity of the Angle CAB.*

As the Baſe AB is to the Perpendicular BC, ſo is the Radius to the Tangent of the Angle BAC ſought.

To do this by the Sector, take 20 from your Line of Lines, repreſented by 2; then ſet this over as a Parallel on 6 and 6, repreſenting 60, on the ſaid Lines of Lines. The Sector remaining thus open'd, take the parallel Radius, that is, the Extent from 45 Degrees to 45 Degrees on the Lines of Tangents; and this laterally meaſur'd on either of the Lines of Tangents, will give about 18 Degrees 26 Minutes, for the Quantity of the ſought Angle CAB.

Note, If the parallel Radius be longer than either of the Lines of Tangents, which it will often be, then you muſt make uſe of the Lines of leſſer Tangents. But theſe leſſer Tangents muſt make the ſame Angle with each other as the Lines of Lines do, as I mentioned before in the Deſcription of the Sector; for otherwiſe the Proportion cannot be worked, becauſe ſimilar Triangles are requir'd.

PROB. IV.

The Hypothenuſe AC, and the Angle ACB, of the right angled ſpherical Triangle ABC, being given, the one 50 Degrees, and the other 23 Degrees 30 Minutes; to find the Side AB. *Fig. 20.*

As Radius is to the Sine of the Hypothenuſe, ſo is the Sine of the Angle ACB to the Sine of the ſought Side AB.

To do this by the Sector, take the lateral Extent of 50 Degrees on the Sines between your Compaſſes; then open the Sector, and ſet over this Extent on 90 and 90 of the Lines of Sines. The Sector remaining thus opened, take the parallel Extent of 23 Degrees 30 Minutes, on the Sines, which laterally meaſur'd on either of them, will give the Sine of about $17\frac{2}{3}$ Degrees, the ſought Side AB.

PROB. V.

The Angles BCA, BAC, and the Side Fig. 21. *BC of an oblique ſpherical Triangle, being given; the firſt 20 Degrees, the ſecond 30 Degrees, and the laſt 25 Degrees; to find the Side AB.*

As the Sine of the Angle BAC is to the Sine of the Side BC, ſo is the Sine of the Angle BCA to the Sine of the Side AB ſought.

To do this by the Sector; open it, and ſet the Sine of 25 Degrees over as a Parallel on the Sines of 30 and 30; then the Sector remaining thus opened, take the parallel Sine of 20 Degrees, and meaſure it laterally on the Sines, and you will find it almoſt 17 Degrees 50 Minutes, which will be the Quantity of the ſought Side AB.

PROB. VI.

The Side AC, and the Angle BAC of Fig. 22. *the right angled ſpherical Triangle ABC being given; the one 27 Degrees 54 Minutes, and the other 23 Degrees 30 Minutes; to find the Side BC.*

As Radius is to the Sine of the given Side

AC; ſo is the Tangent of the given Angle BAC, to the Tangent of the required Side BC.

To do this by the Sector; open it, and ſet over the Sine of 27 Degrees 54 Minutes, as a Parallel on 90 and 90 of the Lines of Sines. The Sector remaining thus opened, take the parallel Tangent of 23 Degrees 30 Minutes, and meaſuring it laterally on either of the Lines of Tangents, you will find it about 11 Degrees 30 Minutes, which is the Quantity of the ſought Side BC.

Theſe few Examples being well underſtood, any one may of himſelf find out the manner of working any other of the Caſes of plain or ſpherical Trigonometry, by the Sector. Therefore I now proceed to ſhew the excellent Uſe of this Inſtrument, in drawing the Hour Lines upon an Horizontal, and erect South Dial.

PROB. VIII.

To draw the Hour Lines upon an Horizontal and Vertical Plan.

First, draw a right Line AC for the Horizon and Equator, and croſs it at the Point A, about the middle of the Line, with AB another right Line, which may ſerve for the Meridian and Hour Line of 12; then take out 15 Degrees from your Tangents, and lay them off in the Equator on both ſides from 12; one of which Points will ſerve for the Hour of 11, and the other for the Hour of 1. Again take out the *Fig.* 23.

Tangent of 30 Degrees, and prick it down in the Equator on both ſides from 12; one of which Points will ſerve for the Hour of 10, and the other for the Hour of 2. In like manner prick down the Tangent of 45 Degrees for the Hours of 9 and 3, and the Tangent of 60 Degrees for the Hours of 8 and 4, and the Tangent of 75 Degrees for the Hours of 7 and 5.

In like manner may the Parts of an Hour be pricked down, in allowing 7 Degrees 30 Minutes for every half Hour, and 3 Degrees 45 Minutes for every Quarter. This done, you are to conſider the Latitude of the Place, and the Quality of the Plan; for the Secant of the Latitude will be the Semidiameter in a vertical Plan, and the Secant of the Complement of the Latitude in an horizontal Plan.

For Example; About *London* the Latitude is 51 Degrees 32 Minutes, and let the Plan be vertical. If you take AV, the Secant of 51 Degrees 32 Minutes from the Sector, having firſt opend the Sector ſo, that the Radius of the Lines of Secants be equal to the Radius of the Tangents pricked off, and prick it down in the meridian Line from A to V; the Point V will be the Center of the Dial: And if you draw right Lines from V to 11, and 10, and the reſt of the Hour Points, theſe will be the Hour Lines required.

But if the Plan be an Horizontal one, you muſt take out AH, the Secant of 38 Degrees 28 Minutes, and prick it down in the Meridian

Line from A to H; ſo right Lines drawn from the Center H to the Hour Points, will be the Hour Lines required. Only the Hour of *6* is wanting, which muſt always be drawn parallel to the Equator, thro' the Center V, in a vertical Plan; and thro' the Center H, in an horizontal Plan.

But if it happen that ſome of the Hour Points fall out of your Plan, you may remedy your ſelf, both in the vertical and horizontal Plans.

For if at the Hour Points of 3 and 9 you draw occult Lines parallel to the Meridian, the Diſtances DC, between the Hour Line of *6*, and the Hour Points of 3 and *9*, will be equal to the Semidiameter AV, in a vertical, and AH, in an horizontal Plan; and if they be divided in the ſame manner that the Line AC is divided, you will have the Points of 4, 5, 7, and 8, with their Halves and Quarters.

As in the horizontal Plan, take out the Semidiameter AH, and make it a parallel Radius, by ſetting it over in the Sines of *9*0 and *9*0: Then take the parallel Tangent of 15 Degrees, and it will give you the Diſtance from *6* to 5, and from *6* to 7, in your horizontal Plan. The Sector remaining thus opened, take out the parallel Tangent of 30 Degrees, and it will give you the Diſtance from *6* to 4, and from *6* to 8, in your horizontal Plan. The like may be done for half and quarters of Hours.

CHAP. VI.

Of the Uſe of the Lines of Numbers, artificial Sines, and Tangents.

SECTION I.

Of the Uſe of the Line of Numbers.

THE Order of the Diviſions on the Line of Numbers is thus; it begins with 1, and ſo proceeds with 2, 3, 4, 5, 6, 7, 8, 9; and then 1, 2, 3, *&c.* to 10 at the end. Each of theſe Diviſions are ſubdivided into 10 Parts, and every of theſe 10*th* Parts from 1 in the middle to 2, are again ſubdivided into 5 Parts. But the 10*th*'s from the ſecond 2 to 5, are only ſubdivided into two Parts.

Note, Each 10*th* ſhould have been ſubdivided into 10 other Parts, called Centeſmes, but that the Length of the Line will not admit of it.

Numeration on the Line of Numbers.

PROB. I.

Any whole Number under four Figures being given; to find the Point on the Line of Numbers repreſenting the ſame.

Firſt ſeek for the firſt Figure of your Number amongſt the long Diviſions that are figured, and that leads you to the firſt Figure of the Number. Then for the ſecond Figure, count ſo many Tenths from the aforeſaid Diviſion forwards, as the ſecond Figures amount to; then for the third Figure, count from the laſt Tenth, as many Conteſmes as the third Figure contains; and for the fourth Figure, count from the laſt Centeſme, ſo many Milions, as the fourth Figure hath Unites; and that will be the Point where the Number is on the Line of Numbers.

Example; To find the Point on the Line of Numbers repreſenting the Number 12. Now becauſe the firſt Figure of this Number is 1, you muſt take the 1 in the middle for the firſt Figure; then the next Figure being 2, count the Tenths from that 1, and *there* will be the Point repreſenting 12.

Again; To find the Point repreſenting 144. Firſt, as before, take for 1, the firſt Figure of the Number 144, the middle Figure 1; then for the ſecond, (*viz.* 4) count 4 Tenths forwards; laſtly for the other 4, count 4 Centeſmes further, and that is the Point for 144.

Laſtly; To find the Point repreſenting 1728. Firſt, as before, for 1000 take the middle 1; ſecondly for 700 reckon 7 Tenths forwards; thirdly for 20 reckon 2 Centeſmes from that ſeventh Tenth; and laſtly, for the 8 you muſt reaſonably eſtimate the following Centeſme to be ſubdivided into 10 Parts, and 8 of them will be the Point on the Line of Numbers repreſenting 1728. Underſtand the ſame for any other Number. The aforeſaid Point may alſo repreſent .1728, or 1.728, or 17.28, or laſtly 172.8.

PROB.

PROB. II.

One Number being given to be multiply'd by another; to find the Product.

Extend your Compaſſes from Unity to the Multiplicator; then the ſame Extent apply'd the ſame way, from the Multiplicand, will cauſe the movable Point to fall upon the Product.

Example; Let 6 be given to be multiply'd by 5; Extend your Compaſſes from 1 to 5, and the ſame Extent will reach from 6 to 30, the Product ſought. Again, ſuppoſe 125 is to be multiply'd by 144: Extend your Compaſſes from 1 to 125, and the movable Point will fall from 144 upon the Product 18000.

PROB. III.

One Number being given to be divided by another; to find the Quotient.

Extend your Compaſſes from the Diviſor to 1, and the ſame Extent will reach from the Dividend to the Quotient. Or, extend the Compaſſes from the Diviſor to the Dividend, the ſame Extent will reach the ſame way from 1 to the Quotient.

Example; Let 750 be a Number given to be divided by 25. Extend you Compaſſes downwards from 25 to 1; then applying that Extent the ſame way, from 750, and the other Point will fall upon 30, the Quotient ſought.

PROB. IV.

Three Numbers being given, to find a fourth Number proportional to them.

Extend your Compaſſes from the firſt of the given Numbers to the ſecond [illegible] the ſame Denomination; and this Extent apply'd the ſame way, will reach from the ſecond to the fourth Proportional ſought.

Example; If 8 give 16, what will 12 give? Extend your Compaſſes from 8 to 16; and this Extent apply'd the ſame way will reach from 12 to 24, the fourth Proportional ſought.

Again, If 38 gives 76, what will *96* give? Extend your Compaſſes from 38 to *96*, and the ſame Extent will reach from *76* to *192*, the fourth Proportional ſought.

PROB. V.

To find a mean Proportional between two given Numbers.

Biſect the Diſtance between the given Numbers; and that Point of Biſection will fall on the mean Proportional ſought.

Example; The Extremes being 8 and 32, the middle Point between them will be 16, the mean Proportional ſought. For as 8 is to 16, ſo is 16 to 32.

PROB. VI.

To find the Square Root of a given Number.

The ſquare Root of any Number is always a mean Proportional between 1 and the ſaid

Number; ſo that to find the ſquare Root of a Number is only to take half the Diſtance between 1 and the Number given, and that Point of Biſection will be the ſquare Root of the Number given. But if the Figures of the Number be even, you muſt look for the Unite at the beginning of the Line, and the Number in the ſecond Part, and the Root in the firſt Part. Or rather rec[illegible] 10 at the end to be Unity, and then both Root and Square will fall backwards towards the middle, in the ſecond Part of the Line. But if the Number be odd, then the middle Unite will be moſt convenient, and both Root and Square will be found from thence forwards towards 10.

SECTION II.

Of the conjoint Uſe of the Lines of Numbers, artificial Sines, and Tangents.

Theſe Lines of artificial Numbers, Sines, and Tangents, are ſo contrived, and ſet to each other, that by means of a Pair of Compaſſes, any Problem, whether in right lined or ſpherical Trigonometry, may be very expeditiouſly ſolved by them with tolerable Exactneſs.

The Sines are numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90, ſignifying Degrees. Every one of the Degrees, as far as 6, are ſubdivided into 10 Parts, ſignifying every 6 Minutes; then the Degrees from 6 to 10, are only ſubdivided into 6 Parts for every 10 Minutes. Again, each of the 10 Degrees, which have not Numbers ſet to them, for avoiding Confuſion, between 10 and 20, are only divided into 4 Parts, for every 15 Minutes; the Degrees between 20 and 30, are divided into 3 Parts, for every 20 Degrees; the Degrees from 30 to 50, are halved, for every 30 Minutes; after which to 70 there are only Degrees, and from 70 to 80 there are only every two Degrees repreſented; and from 80 to 90 none.

The Line of artificial Tangents, is number'd 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 45. To the Tangents 10, 20, 30, 40, are ſet their Complements 80, 70, 60, 50. Alſo theſe ſignify Degrees, like as the Sines.

Theſe Degrees are each ſubdivided, to the Tangent of 30 Degrees, into the ſame Number of Parts, as thoſe on the artificial Sines. Each of which repreſent the ſame Numbers of Minutes as thoſe on the Sines. But from 30 Degrees to 45, each Degree is ſubdivided into 3 Parts, for every 20 Minutes.

PROB. I.

To ſolve the Caſe and Example of Prob. I. of the laſt Chapter.

Set one Foot of your Compaſſes upon 7 of the Line of Numbers, repreſenting 70 Miles, and extend the other to 40 Miles, repreſented by 4. The Compaſſes remaining thus opened, ſet one Foot upon 90 of the Line of Sines, and the other Foot will fall downwards upon the Line of Sines, on the Sine of 34 Degrees 50 Minutes, the Quantity of the Angle ſought.

PROB. II.

To ſolve the Caſe and Example of Prob. II. of the laſt Chapter.

Set one Foot of your Compaſſes upon the Sine of 30 Degrees, and extend the other to 60 Leagues on the Line of Numbers: The Compaſſes remaining thus opened, ſet one Foot upon the Sine of 46 Degrees, and the other will fall upwards on the Line of Numbers upon 86½ Leagues, the Length of the ſought Side.

PROB. III.

To ſolve the Caſe and Example of Prob. III. of the laſt Chapter.

Set one Foot of your Compaſſes on 60 of the Line of Numbers, and extend the other to 45 Degrees of the Line of Tangents: The Compaſſes remaining thus opened, ſet one Foot upon the Number 20 on the Line of Numbers, and the other will fall on the Line of Tangents upon the Tangent of about 18 Degrees 26 Minutes, the Quantity of the Angle ſought.

PROB. IV.

To ſolve the Caſe and Example of Prob. IV. of the laſt Chapter.

Set one Foot of your Compaſſes upon the Sine of 50 Degrees, and extend the other to the Radius or Sine of 90 Degrees; the Compaſſes remaining thus opened, ſet one Foot upon the Sine of 23 Degrees 30 Minutes, and the other will fall downward upon the Sine of 17 Degrees, and about 3 Fourths, for the Length of the ſought Side.

PROB. V.

To ſolve the Caſe and Example of Prob. V. of the laſt Chapter.

Set one Foot of your Compaſſes upon the Sine of 30 Degrees, and extend the other downwards to the Sine of 25 Degrees: The Compaſſes remaining thus opened, ſet one Foot upon the Sine of 20 Degrees, and the other will fall upon the Sine of about 16 Degrees, 40 Minutes, the Length of the ſought Side.

PROB. VI.

To ſolve the Caſe and Example of Prob. VI. of the laſt Chapter.

Set one Foot of your Compaſſes upon the Sine of 90 Degrees, and extend the other downwards to the Sine of 27 Degrees, 54 Minutes: Your Compaſſes remaining thus opened, ſet one Foot upon the Tangent of 23 Degrees, 30 Minutes, and the other Foot will fall downwards upon the Tangent of about 11½ Degrees, the Quantity of the ſought Side.

Note, If in working Proportions in Sines and Tangents, the Foot of your Compaſſes ſhould fall beyond the Tangent of 45 Degrees, as it will often happen, then you muſt take a Point as far on this Side of 45 Degrees, as the ſaid Point of your Compaſſes falls beyond 45 Degrees, and that Point will give you the Tangent of the Complement to 90 Degrees, of the Side or Angle ſought.

Example; Suppoſe this Proportion was to be worked as the Sine of 20 Degrees is to Radius; ſo is the Tangent of 30 Degrees to another Tangent; ſet one Foot of your Compaſſes on the Sine of 20 Degrees, and extend the other to 90 Degrees: Your Compaſſes remaining thus opened, ſet one Foot upon the Tangent of 30 Degrees, and the other will fall upwards beyond the Tangent of 45 Degrees; then finding how far it falls beyond, with one Foot of your Compaſſes on 45 Degrees, ſet off that Extent on this ſide 45 Degrees, and you will find it to fall upon the Tangent of the Complement of the ſought Tangent, *viz.* on 30 Degrees, 40 Minutes; whence the Tangent ſought is 59 Degrees, 20 Minutes.

Hence you ſee, that in working Proportions in Numbers and Sines, or Sines and Tangents, it is but ſetting one Foot of your Compaſſes upon the Number, Sine, or Tangent, in the firſt Term, and extending the other to the ſecond Term of a contrary Denomination; and then that Extent will reach from the third Term to a fourth ſought.

FINIS.

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